Disjunctive inferences and presupposition projection

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Overview. Plural indefinites and plural definites both give rise to a truth value gap – states of affairs in which neither a sentence nor its negation are judged true. In the case of indefinites, this gap, which we term here *multiplicity*, is manifested in states of affairs in which there is exactly one individual in the extension of the indefinite for which the predicate is true (1). In the case of definites, the gap is traditionally termed *homogeneity*, and is manifested in states of affairs where the predicate is true for some but not all individuals in the extension of the definite (2). Despite the obvious similarity between these two phenomena, they are often discussed and analyzed in the literature in different terms. In this talk, we propose that treating them as essentially the same phenomenon can provide us with a parsimonious account of both, while explaining some of their distinctive properties.

Theories of multiplicity and homogeneity. The different accounts proposed in the literature on multiplicity and homogeneity can be roughly divided into two classes: (i) *asymmetric theories*, which posit that the reason for the gap is some covert semantic operation that is active in matrix sentences but not under negation (Sauerland 2003, Spector 2007, Zweig 2008, Ivlieva 2013, Bar Lev 2020, a.o.); (ii) *disjunctive theories*, which maintain that the the gap is the result of a disjunctive component in the meaning of these sentences. This includes accounts that argue that bare/definite plurals are ambiguous between strong and weak readings (Krifka 1996, Magri 2014, Križ and Spector 2017, a.o.), and accounts that argue for a disjunctive presupposition (Schwarzschild 1994, Gajevski 2005, Križ and Chemla 2015, Križ 2017, Wehbe 2022, a.o.). A problem with the asymmetric theories is that they predict that no inferences should be generated under negation. As we show in the next section, this does not seem to be the case, both for multiplicity and for homogeneity. On the other hand, the disjunctive component in the second class of theories is generally based on a stipulation about the semantics of plural (in)definites. Here, we propose an account which avoids both of these problems – we will adopt the claim that the gaps are the result of a disjunctive presupposition, but unlike previous accounts, cash out this idea in an explanatory way, as the result of an interaction between a local application of the operator *pex* (Bassi et al. 2021), and *Strong Kleene* presupposition projection (George, 2008).

The presuppositional nature of multiplicity and homogeneity. We argue, following Schwarzschild 1994, Gajevski 2005, Križ and Chemla 2015, Križ 2017, Wehbe 2022 (a.o.), that sentences like (1a) presuppose that either Jack saw multiple horses or he saw no horses (3), and that sentences like (2a) presuppose that either Mary read all of the (contextually salient) books or she read none of them (4). Given that presuppositions project from under negation, it follows immediately that the negated sentences in (1b) and (2b) carry the same presuppositions (5)-(6). Wehbe (2022) provides an indirect argument for the above semantics for homogeneity based on the principle in (7), termed Post-Accommodation Informativity (PAI) by Doron and Wehbe (2022). He shows that a matrix sentence containing a definite plural is infelicitous whenever the context is set up such that it is common ground that the predicate is true for at least some of the individuals in the extension of the definite. This follows from the semantics in (4) in combination with PAI, since accommodating the disjunctive presupposition in such a common ground results in a context which entails that Mary read all of the books, rendering the assertive component trivial. Wehbe further shows that a negated sentence containing a definite plural is infelicitous whenever it is common ground that the predicate cannot be true for all individuals denoted by the definite. This is explained by the semantics in (6) in a similar way: accommodating the *all-or-none* presupposition yields a common ground which entails that Mary read none of the books, resulting in triviality. Notice that this is not predicted by asymmetric theories, which maintain that no inference arises in negated sentences. We argue here that the same method can be used to show that multiplicity is a disjunctive presupposition as well. Consider the example in (8), where it is common ground that Jack won either one or three books. The use of the indefinite plural is infelicitous here, for a similar reason as in Wehbe's cases: accommodating the presupposition that Jack either won multiple books or no books results in a common ground which entails that Jack won three books, rendering the sentence trivial. The generalization that follows is that the use of plural indefinites in matrix sentences is blocked in a common ground which entails that the predicate is true for at least one individual in the set denoted by the noun. And again, the flip-side is true for negated sentences containing plural indefinites – they are infelicitous whenever it is common ground that the predicate is true for at most one individual in the extension of the noun. This is evidenced by examples like (9), assuming that it is common ground that cars cannot have multiple steering wheels[1]. We conclude that both multiplicity and homogeneity can be detected even under negation, and behave like presuppositions in the sense that they must be accommodated in the common ground. The question that arises is where do these presuppositions come from, and what is responsible for their disjunctive form.

Deriving the multiplicity presupposition. We argue that the source of both multiplicity and homogeneity is a local application of the operator *pex* (Bassi et al. 2021). A slightly simplified version of its basic definition is given in (10)-(11) (note that we do not assume Innocent Inclusion). Let us consider first multiplicity, where we assume the indefinite is existentially closed. Following Mayr 2015, we posit that *pex* is type-flexible, and applies directly to the noun, below the existential quantifier (the <et,et> version of *pex* is defined in (12)). We end up with the LF in (13) for the basic sentence (1a) (ignoring possible appearances of *pex* in other positions, as they would not affect the meaning here). Finally, we assume that the plural and singular form are each other's alternatives, and that they have the semantics in (14) for number marking, following the standard assumption in the literature. It is easy to see that *pex* adds the presupposition to the NP *horses* that the set it denotes does not contain atomic horses. Since this presupposition is triggered in the restrictor of an existential quantifier, it is not obvious how it should project to the entire sentence. While there are conceptual arguments to be made here, we observe that adopting a Strong Kleene theory of projection (George, 2008) gives us the correct empirical result. To see why that is the case, let us analyze the abstract case: a proposition of the form $\exists x$ $[P_\pi(x) \wedge Q(x)]$ (where P presupposes π). Notice that the proposition is true for any bivalent correction of P iff there exists an individual a such that $\pi(a) \wedge P_\pi(a) \wedge Q(a)$, and it is false for any bivalent correction iff for every individual a, either $\neg Q(a)$ or $\pi(a) \wedge \neg P_\pi(a)$. We conclude that the entire proposition should presuppose $(\exists x\, [\pi(x) \wedge P_\pi(x) \wedge Q(x)]) \vee (\forall x\, [Q(x) \rightarrow (\pi(x) \wedge \neg P(x))])$. Applying our conclusion to the LF in (13), we end up correctly predicting the truth conditions in (3)[2]. Notice that we have derived the disjunctive from of the presupposition directly from the logical structure of the sentence (along with some independently-motivated assumptions about the scalar implicatures and presupposition projection), without appealing to stipulations.

Deriving the homogeneity presupposition. We argue that a very similar analysis might be

[1]Curiously, this kind of examples are already discussed in Spector 2007, where he mentions them as open issues for a theory of multiplicity. They are also discussed by Ahn et al. (2020), who try to account for them using *Maximize Presupposition!*.

[2]We remain agnostic about the mechanism which allows distributive predicates like *see* to apply to plural individuals, while keeping to the standard assumption that the result is a universal distribution of the predicate over the atoms of the plural individual.

used to explain the homogeneity presupposition to which definite plurals give rise. We note that a version of *pex* suggested in Del Pinal et al. 2024 can derive this presupposition straightforwardly, by hard-coding into the definition of *pex* a disjunctive notion of Innocent Inclusion. They propose to add to the definition in (11) the presupposition that either all innocently includable alternatives are true, or they are all false. When plugged into Bar Lev's (2020) account of homomgeneity, this derives the desired presupposition. Notice however that this way essentially stipulates the disjunctive form of the homogeneity presupposition. We propose here that as in the case of multiplicity, the disjunctive nature of the homogeneity presupposition is a result of the logical properties of the environment in which it is triggered. Our account is based on the idea from Magri 2014 that the core semantics of definites is existential. We further assume, following Malamud 2012 and Bar Lev 2020, that definites give rise to sub-domain alternatives – alternatives in which the noun in the scope of the definite article is replaced with its subsets (we remain agnostic about the mechanism which is responsible for those alternatives). In fact, we can assume that the existence of subdomain alternatives is the only semantic difference between definites and indefinites. Finally, we assume that each of those alternatives is closed under sum-formation. These assumtpions are layed out in (15). As in the case of indefinites, a *pex* operator applies directly to the NP in sentences like (2a), whose LF is given in (16). Notice that every subdomain alternative of [book PL] (beside the one equivalent to the NP itself) is innocently excludable. The result of applying *pex* below the definite article is thus adding the presupposition to the NP [book PL] that the set it denotes does not contain any book-individual apart from the maximal book-plurality. That is because the maximal book-plurality is the only individual which is an element of [book PL] but not of any of its excludable alternatives. The projection of this presupposition is now identical to the case of multiplicity: assuming Strong Kleene, the LF in (16) will be true if Mary read the maximal book-plurality, and false if she did not read any book-individual. Those are indeed the truth conditions in (4) for which we argued. Again, we have cashed out these truth conditions without stipulating any special lexical feature. Admittedly, the case of definites demanded us to diverge significantly from standard assumptions about their semantics. In this talk, we will examine the consequences of this divergence in more detail, and provide further evidence that it is on the right track.

Examples

(1) a. Jack saw horses. $\rightsquigarrow$ Jack saw more than one horse.
b. Jack didn't see horses. $\rightsquigarrow$ Jack saw no horses. (Spector, 2007)

(2) a. Mary read the books. $\rightsquigarrow$ Mary read all of the books.
b. Mary didn't read the books. $\rightsquigarrow$ Mary read none of the books.

(3) $$[\![\text{Jack saw horses}]\!] = \begin{cases} 1 & \text{if Jack saw multiple horses} \\ 0 & \text{if Jack saw no horse} \\ \# & \text{otherwise} \end{cases}$$

(4) $$[\![\text{Mary read the books}]\!] = \begin{cases} 1 & \text{if Mary read all the books} \\ 0 & \text{if Mary read non of the books} \\ \# & \text{otherwise} \end{cases}$$

(5) $$[\![\text{Jack didn't see horses}]\!] = \begin{cases} 1 & \text{if Jack saw no horses} \\ 0 & \text{if Jack saw multiple horses} \\ \# & \text{otherwise} \end{cases}$$

(6) $[\![$Mary didn't read the books$]\!]$ = $\begin{cases} 1 & \text{if Mary read none the books} \\ 0 & \text{if Mary read all of the books} \\ \# & \text{otherwise} \end{cases}$

(7) **Post-Accommodation Informativity:** A sentence S presupposing p can be uttered felicitously only if it is not trivial with respect to the context set after presupposition accommodation. (Doron and Wehbe 2022)

(8) **Context:** Jack participates in a competition where the first prize is three books and the second prize is one book. Jack was just announced as one of the winners, but we're not sure which prize he got. Mary goes to check and says:
a. He won several books!
b. #He won books!

(9) a. This autonomous car doesn't have a steering wheel.
b. #This autonomous car doesn't have steering wheels. (Adapted from Spector 2007)

(10) $IE(\phi) = \{\psi |\ \psi \in Alt(\phi) \wedge [\![\phi]\!] \not\subseteq [\![\psi]\!]\}$

(11) $[\![pex\ \phi]\!] = \begin{cases} 1 & \text{if } [\![\phi]\!] = 1 \bigwedge\{[\![\psi]\!] = 0 : \psi \in IE(\phi)\} \\ 0 & \text{if } [\![\phi]\!] = 0 \\ \# & \text{otherwise} \end{cases}$

(12) $[\![pex^{et}\ \pi]\!] = \begin{cases} 1 & \text{if } [\![\phi]\!] = 1 \bigwedge\{[\![\psi]\!] = 0 : \psi \in IE(\phi)\} \\ 0 & \text{if } [\![\phi]\!] = 0 \\ \# & \text{otherwise} \end{cases}$

(13) [$\exists$ [pex [horse PL]]λx [Jack saw x]]

(14) a. $[\![$P SG$]\!] = \lambda x.x \in *P \wedge Atom(x)$
b. $[\![$P PL$]\!] = \lambda x.x \in *P$

(15) a. $[\![$the P$]\!] = \lambda Q.\exists x[x \in P \wedge x \in Q]$
b. Alt([the [P PL]]) = $\{*P' | P' \subseteq P\}$

(16) [the [*pex* [book PL]]λx [Mary read x]]

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